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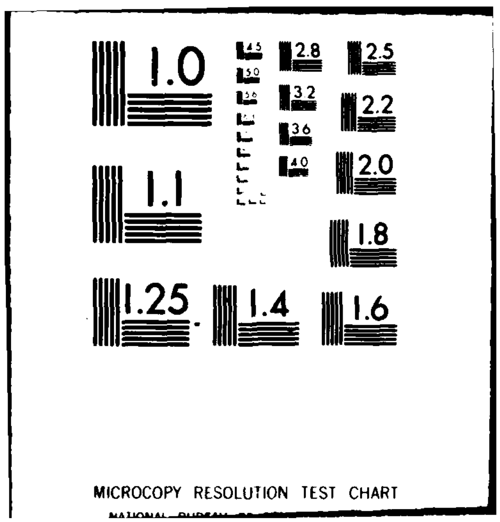
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Technical Report #63

EVALUATION OF PRECEDENCE CRITERIA FOR PROJECT SCHEDULING UNDER RESOURCE CONSTRAINTS

by

Shi Min Chang and Robert L. Sielken, Jr.

Texas A&M University
Office of Naval Research
Contract N00014-78-C-0426
Project NR047-179

ATTACHMENT I

EVALUATION OF PRECEDENCE CRITERIA FOR PROJECT SCHEDULING UNDER RESOURCE CONSTRAINTS

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THEMIS OPTIMIZATION RESEARCH PROGRAM
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INSTITUTE OF STATISTICS
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ATTACHMENT II

ABSTRACT

When more than one activity in a project requires the same indivisible resource, the project schedule can resolve this resource constraint by specifying the order in which these activities are to be performed. Several heuristic criteria for determining this order are considered. The minimum cost schedule for a given project deadline can often be found by determining the optimal schedule ignoring the resource constraints and then resolving any resource usage conflicts by ordering the conflicting activities so as to minimize the project completion time. A simple procedure for determining this latter order is given.

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I. Introduction.

Usually project scheduling techniques consider the project as a network of activities. In the project network an activity can be represented by a branch (arrow) between two nodes (circles). Nodes represent the beginnings and/or completions of activities. For example, the network for a project consisting of activities A, B, C, D, E, F, and G is given in Figure 1.

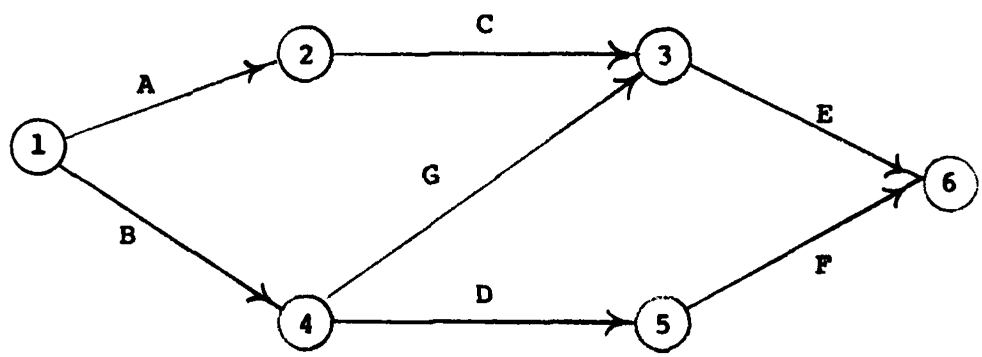

Figure 1. Project Network

The node numbers within the circles are for reference purposes only.

In Figure 1, these six activities have the following relationships:

(i) A and B start at the same time.

(ii) A must be completed before C can be started; and C and G must be completed before E can be started.

B must be completed before D and G can be started; and D must be completed before F can be started. That is, A precedes C, C precedes E, B precedes D, B precedes G, G precedes E, and D precedes F.

(iii) The project will not be finished until both E and F are completed

The time that it actually takes to complete an activity from beginning to end is called the activity's duration and may be a random variable. The mean duration of each activity has been determined for this example and they are given in Table 1.

Table 1. Activity Mean Durations.

Activity	Mean Duration
A	10 hours
B	12 hours
C	8 hours
D	7 hours
E	11 hours
F	9 hours
G	0 hours

The overall project duration has been calculated as the maximum path length where a path length is calculated by the addition of all activity durations along the path. When each activity duration is its mean, the path through

A-C-E takes twenty-eight hours, the path through B-G-E takes twenty-three hours, the path through B-D-F takes twenty-nine hours, and the critical (longest) path is through B-D-E taking twenty-nine hours.

The cost of an activity is assumed to be a convex piecewise linear function of the activity's mean duration. This allows for the possibility that an activity duration can sometimes be shortened by the application of greater amounts of resources such as labor and capital. This implies that the expenditure of more money may reduce the duration of an activity. There is, therefore, a time/cost trade-off for each activity in the project and an overall trade-off involving project duration and project expense. For example, in Figure 2 to have a duration in the interval of $\left[T_1, T_2\right]$ might be more expensive than $\left[T_2, T_3\right]$ due to the use of more efficient equipment.

A project schedule is a specification of each activity's mean duration. The time to complete the project is a random variable whose distribution depends on the activity duration distributions. The objective is to determine a minimum cost project schedule such that the mean of the corresponding project completion time distribution is less than or equal to a specified project deadline.

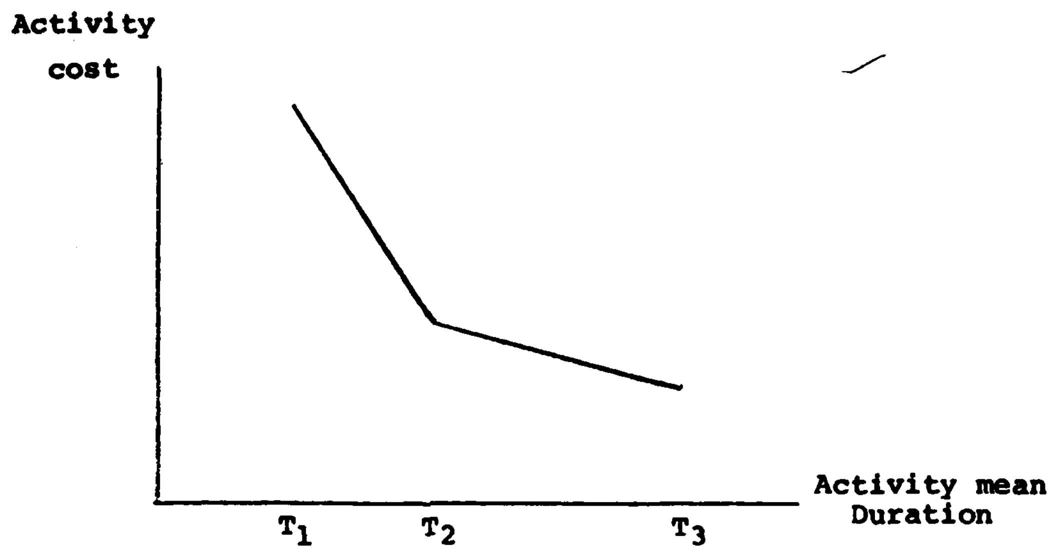

Figure 2. Activity Cost vs. Activity Mean Duration

2. The Problem.

In project scheduling, there are two kinds of feasibility constraints commonly found. First, there are limits on the amount of available resources. Second, there are restrictions on the order in which tasks can be performed. A solution to a project scheduling must be a feasible resolution to both of these two types of constraints. The resolution of the problem of limited resources requires answers to two kinds of questions:

1. Which resources will be allocated to which tasks?
2. When will each activity be performed?

In other words, without the assumption of unlimited resources one must make both allocation decisions and precedence decisions.

For example, in a shipyard there may be only one large capacity crane. Activity A cannot be performed without the crane. The relationship between the mean completion time for activity B, say T_B, and its cost is as in Figure 3.

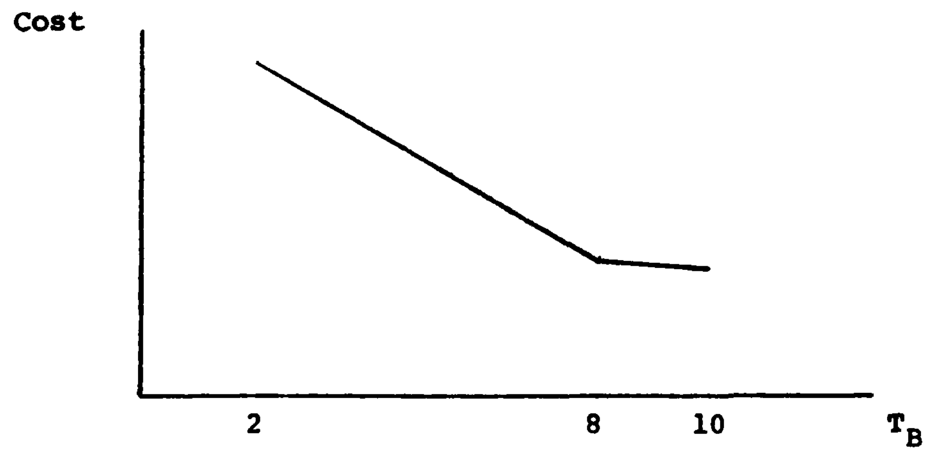

Figure 3. Activity Cost vs. Activity Mean Duration for Activity B

If $2 \leq T_B \leq 8$, the crane must be used for activity B. If $8 \leq T_B \leq 10$, an alternative less expensive procedure not involving the crane is used. Let Start A and Start B be the times at which activities A and B are scheduled to begin. Let T_A (T_B) be the mean duration scheduled for activity A (B). Then there are three possibilities:

1. The time interval [Start A, Start A + T_A] and [Start B, Start B + T_B] do not overlap;
2. the time intervals do overlap but $T_B \geq 8$; and

3. the time intervals overlap and $T_B < 8$.

Only the third possibility causes difficulties for activity A and activity B compete for using the crane.

When we try to resolve the limited resources problem, two reasons stand out as the causes for the current uncertainty and lack of a definitive solution to these problems.

The first difficulty is the statement of the problem. Often times, only a subset of the resources are required for the full duration of the activity; the other resources are needed only for a fraction of that duration. The analyst must then choose between subdividing the activity according to the combination of resources required at any point of time, which may lead to a prohibitively complicated network, or leaving the activity as an entity--a course of action that must lead to a gross exaggeration of the total requirements. Furthermore, it is generally recognized that the time estimates of an activity duration are based on subjective knowledge of the availability of resources. The functional relationship between these two variables may not be known, but it in no sense vitiates its existence. It follows that subdividing the activity in accord with its resources requirement at various points of time may change the time estimates of the subdivisions, which in turn may affect the resource requirement. A vicious cycle results which may disrupt any possibility of obtaining meaningful results.

Another major roadblock to a clear and precise statement of the problem is the fact that an activity can be started and possibly maintained for a long time, with fewer resources than are ideally needed. If the functional relationship between resource availability and the duration were known, analysis similar to the time-cost trade-off analysis could be conducted. But in general, this functional relationship is not defined. Moreover, activities that require more than one resource would require a complete different treatment from those activities requiring only one resource.

The second reason stems from difficulties in the formulation of the mathematical model. There is the interdependence among the activities due to the sharing of the same resources and the dependence of resource consumption on the manner in which the activity is subdivided. Both of these dependencies are basically nonlinear in character. Thus even if the functional relationship describing the dependencies were determined there would still remain the task of combining these individual relations into a meaningful whole.

Furthermore it is often difficult to ascertain the objectives of management, particularly when these objectives are poorly formulated and far from crystallized in management's own mind. For example, the objective may be any one of the following three:

1. Minimize total project cost assuming unlimited availability of the various resources at a price. The price may or may not be linear with the quantity ordered.
2. Minimize the duration of the project under limited availability of resources.
3. Level the resource consumption while meeting a specified project completion deadline.

3. A Heuristic Procedure Based on Precedence Relationship.

If the minimum cost schedule is determined, the situation might be as Figure 4 where activities A, B, and C compete for the same critical resource.

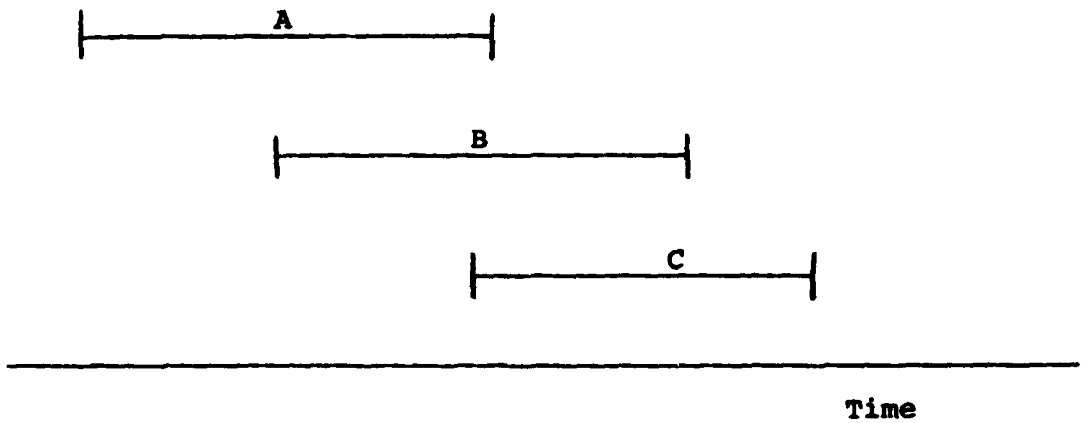

Figure 4. The Time Duration Intervals for Competing Activities

One way to resolve the resource conflict is through required precedence relationships. What we have to do is determine some way of deciding what the precedence relationship among these activities should be; say A before B before C, or B before A before C, etc. There are several reasonable ways of deciding these precedence relationships. For example, one might say that A should start before B if in the current infeasible project schedule

1. activity A started before activity B;
2. the midpoint of the duration interval for activity A was less than that for activity B;
3. activity A finished before activity B;
4. doing activity A before activity B increased the project completion time less than doing activity B before activity A.

5. the cost of decreasing the mean completion time for activity A is less than the corresponding cost for activity B.

Once a precedence relationship among the competing activities has been determined, the project network can be redrawn and a new minimum cost schedule determined.

The whole heuristic procedure to mediate the competition for limited resources by using precedence relationships can be outlined as follows:

Step (1) Determine the minimum cost schedule for the project network disregarding resource constraints. This can be done using the network-flow algorithm described in Technical Report #55 if each activity's duration is assumed to equal its mean duration. Alternatively, it can be done using the Statistical PERT algorithm described in Technical Report #57 if the randomness of each activity's duration is to be explicitly considered.

Step (2) Examine the current project scheduling. If there are no resource conflicts go to Step 4. Otherwise, select a criterion from (1) - (5) as mentioned above for establishing the precedence relationships and determine the precedence relationship associated with each conflict.

Step (3) Redraw the project network to reflect any new precedence relationships. Dummy activities having zero duration and cost may have to be introduced to preserve the origional inter-activity relationships. Return to Step 1.

Step (4) An "optimal" schedule which does not violate the resource constraints has been found.

4. Testing the Precedence Criteria in the Heuristic Procedure.

Since it is impractical to redraw the project network and to explicitly evaluate each precedence criterion (1) - (5) for each project network, the objective is to find guidelines for choosing a precedence criterion which usually provides a near minimum total project cost subject to a specified project deadline and limited critical resources.

To evaluate the precedence criteria (1) - (5), several sample projects have been considered. The first project (P_1) is described in Table 2 and depicted in Figure 5. In Table 2 the activity cost as a convex piecewise linear function of the mean duration is implied by the given time and costs at the ends of the linear segments.

The optimal project schedules (apart from resource constraints) for project deadlines of 39 and 35 are given in Table 3 for project P_1.

Figure 5. Original Project Network for P_1

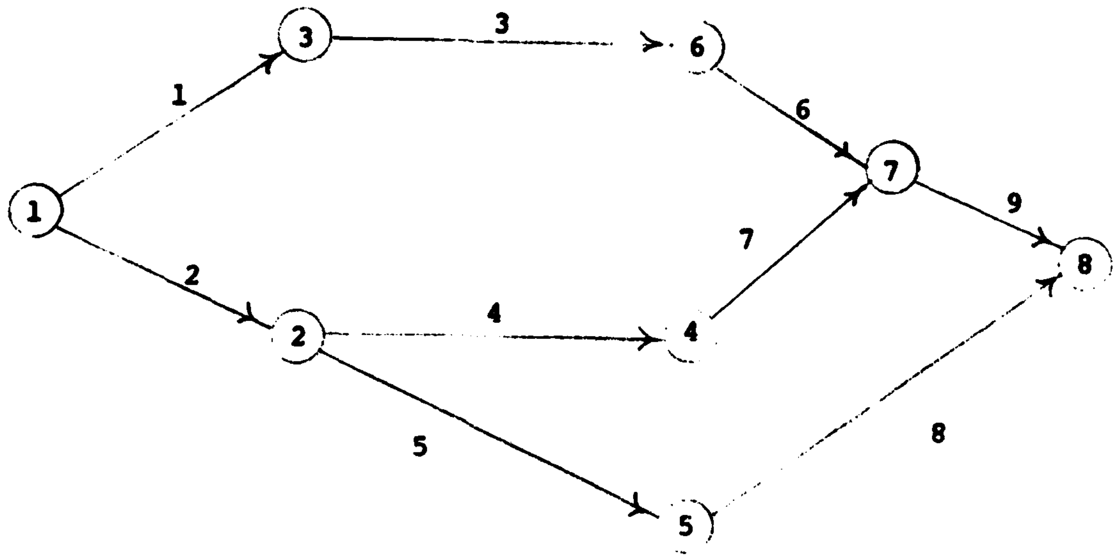

Table 2. Activity Costs and Durations for Project P_1

Activity Number	Origin Node	Terminal Node	Number of Different Completion Times	Time and Cost	Segments		
1	1	3	2	time	4	6	
				cost	8	5	
2	1	2	3	time	3	5	9
				cost	9	6	3
3	3	6	3	time	8	12	15
				cost	15	10	8
4	2	4	3	time	5	8	10
				cost	9	5	3
5	2	5	3	time	4	6	10
				cost	10	6	3
6	6	7	2	time	5	8	
				cost	6	4	
7	4	7	3	time	4	6	7
				cost	8	5	4
8	5	8	2	time	10	15	
				cost	8	4	
9	7	8	2	time	7	10	
				cost	8	4	

Table 3. Optimal Project Schedules
for Two Specified Deadline Times Ignoring
Resource Constraints for Project P_1

Activity Number	Project Deadline 39		Project Deadline 35	
	Activity Duration	Activity Cost	Activity Duration	Activity Cost
1	6	5	6	5
2	9	3	9	3
3	15	8	12	10
4	10	3	10	3
5	10	3	10	3
6	8	4	8	4
7	7	4	7	4
8	15	4	15	4
9	10	4	9	5.33
Project Cost		38		41.33

Suppose that in P_1 activities 3, 4, and 5 compete for the same resources. In what follows the effect on the project cost of using the precedence criteria (1) - (5) to resolve the resources conflict is evaluated.

First, we consider the precedence relationship to be activity 3 before activity 4 before activity 5, ($3 \rightarrow 4 \rightarrow 5$), where "→" implies "precedes". The corresponding project network is in Figure 6. Activity 10 is a dummy activity with zero duration and zero cost. The corresponding optimal schedules are given in Table 4.

Figure 6. Restructured Project Network for $3 \rightarrow 4 \rightarrow 5$

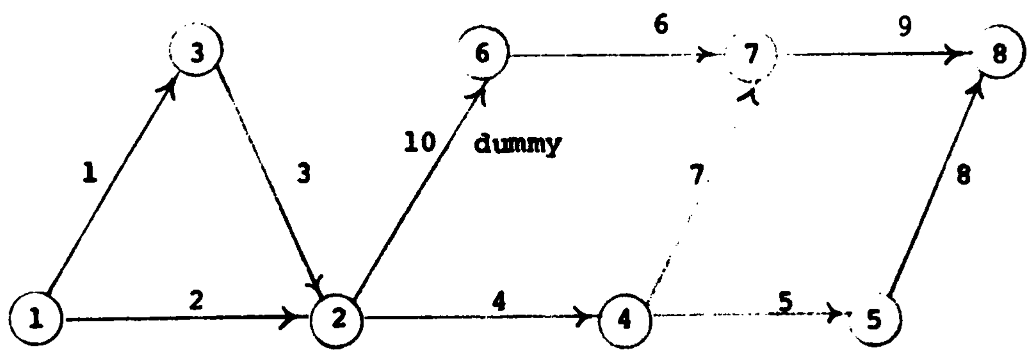

Table 4. Optimal Schedule for 3 → 4 → 5

Activity Number	Project Deadline 39		Project Deadline 35	
	Activity Duration	Activity Cost	Activity Duration	Activity Cost
1	6	5	5	6.5
2	9	3	9	3
3	8	15	8	15
4	8	5	5	9
5	6	6	6	6
6	8	4	8	4
7	7	4	7	4
8	11	7.2	11	7.2
9	10	4	10	4
10	0	0	0	0
Project Cost		53.2		58.7

Second, we consider the precedence relationship activity 3 before activity 5 before activity 4. The corresponding project network and optimal schedules are in Figure 7 and Table 5 respectively.

Figure 7. Restructured Project Network for 3 → 5 → 4

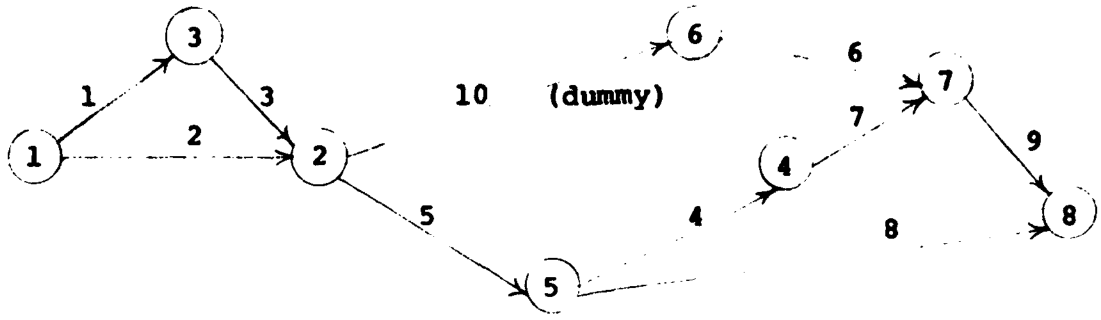

Table 5. Optimal Schedule for 3 → 5 → 4

Activity Number	Project Deadline 39		Project Deadline 35	
	Activity Duration	Activity Cost	Activity Duration	Activity Cost
1	6	5	4	8
2	9	3	9	3
3	8	15	8	15
4	5	9	5	9
5	6	6	6	6
6	8	4	8	4
7	6	5	5	6.5
8	15	4	15	4
9	8	6.67	7	8
10	0	0	0	0
Project Cost		57.67		63.5

Third, we consider the precedence relationship activity 4 before activity 5 before activity 3. The corresponding project network and optimal schedule are in Figure 8 and Table 6 respectively.

Figure 8. Restructured Project Network for 4 → 5 → 3

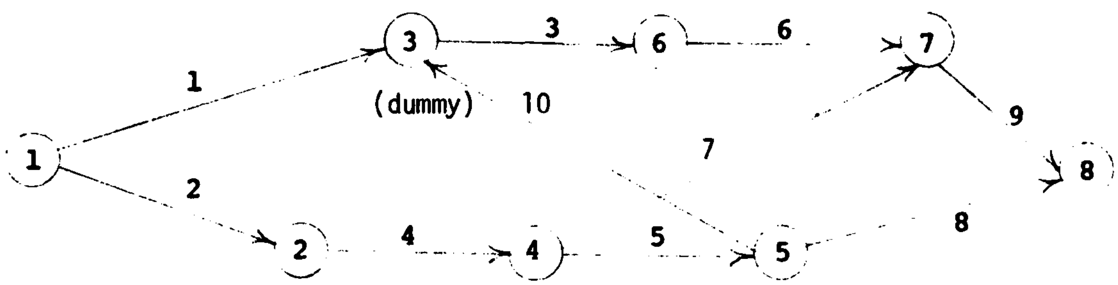

Table 6. Optimal Schedule for $4 \rightarrow 5 \rightarrow 3$

Activity Number	Project Deadline 39		Project Deadline 35	
	Activity Duration	Activity Cost	Activity Duration	Activity Cost
1	6	5	6	5
2	5	6	4	7.5
3	8	15	8	15
4	5	9	5	9
5	6	6	6	6
6	5	6	5	6
7	7	4	7	4
8	15	4	15	4
9	10	4	7	8
10	0	0	0	0
Project Cost		59		64.5

Fourth, we consider the precedence relationship activity 4 before activity 3 before activity 5. The corresponding project network and optimal schedule are in Figure 9 and Table 7 respectively.

Figure 9. Restructured Project Network for 4 → 3 → 5

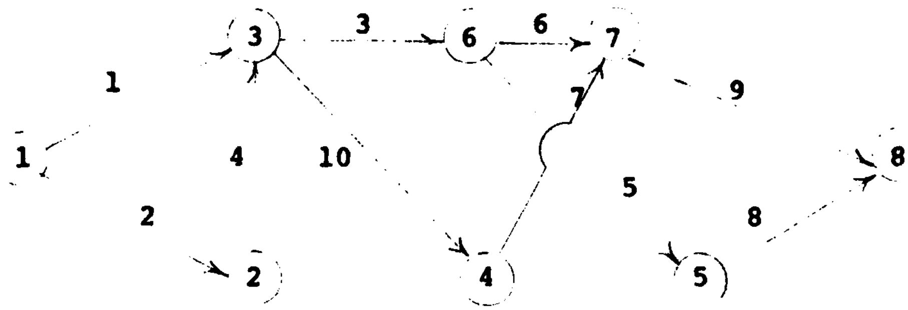

Table 7. Optimal Schedule for 4 → 3 → 5

Activity Number	Project Deadline 39		Project Deadline 35	
	Activity Duration	Activity Cost	Activity Duration	Activity Cost
1	6	5	6	5
2	5	6	5	6
3	8	15	8	15
4	8	5	5	9
5	6	6	6	6
6	8	4	7	4.67
7	7	4	7	4
8	12	6.4	11	7.2
9	10	4	10	4
10	0	0	0	0
Project Cost		55.4		60.87

Fifth, we consider the precedence relationship activity 5 before activity 4 before activity 3. The corresponding project network and optimal schedule are in Figure 10 and Table 8 respectively.

Figure 10. Restructured Project Network for 5 → 4 → 3

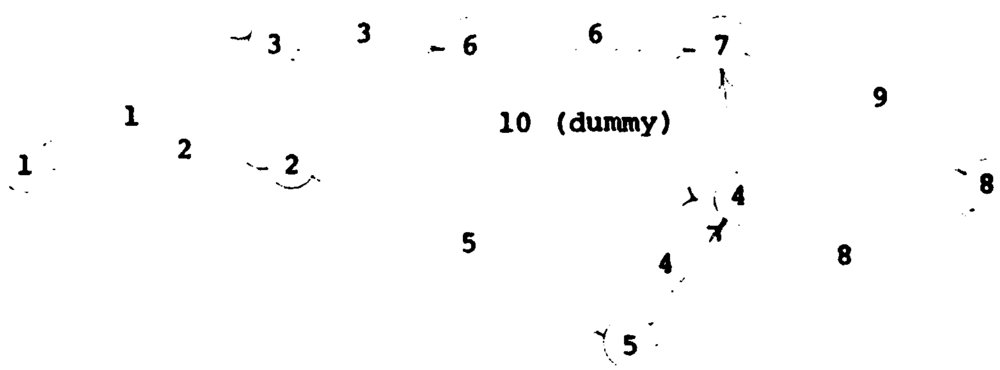

Table 8. Optimal Schedule for $5 \rightarrow 4 \rightarrow 3$

Activity Number	Project Deadline 39		Project Deadline 35	
	Activity Duration	Activity Cost	Activity Duration	Activity Cost
1	6	5	6	5
2	5	6	4	7.5
3	8	15	8	15
4	5	9	5	9
5	6	6	6	6
6	5	6	5	6
7	7	4	7	4
8	15	4	15	4
9	10	4	7	8
10	0	0	0	0
Project Cost		59		64.5

Sixth, we consider the precedence relationship activity 5 before activity 3 before activity 4. The corresponding project network and optimal schedule are in Figure 11 and Table 9 respectively.

Figure 11. Restructured Project Network for 5 → 3 → 4

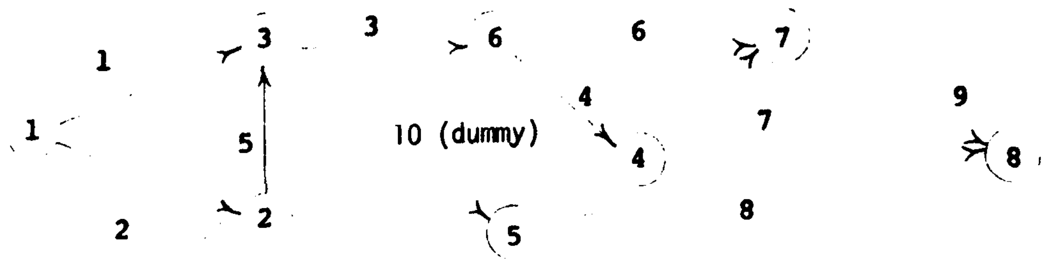

Table 9. Optimal Schedule for 5 → 3 → 4

Activity Number	Project Deadline 39		Project Deadline 35	
	Activity Duration	Activity Cost	Activity Duration	Activity Cost
1	6	5	6	5
2	5	6	3	9
3	8	15	8	15
4	5	9	5	9
5	6	6	6	6
6	8	4	8	4
7	6	5	6	5
8	15	4	15	4
9	9	5.33	7	8
10	0	0	0	0
Project Cost		59.33		65

Table 10 indicates the precedence relationship for project P_1 corresponding to each of the precedence criteria (1) - (5). If a criterion implies two possible precedence relationships, both are listed in Table 10.

Table 10. Precedence Relationships and Project Costs Corresponding to Precedence Criteria (1) - (5).

Project Deadline	Criteria	Precedence Relationship	Project Cost
39	1	3 → 4 → 5	53.20
		3 → 5 → 4	57.67
	2	3 → 4 → 5	53.20
		3 → 5 → 4	57.67
	3	5 → 4 → 3	59
		4 → 5 → 3	59
	4	3 → 4 → 5	53.2
		4 → 5 → 3	59
	5	5 → 4 → 3	59.33
35	1	3 → 4 → 5	58.7
		3 → 5 → 4	63.5
	2	5 → 4 → 3	64.5
		4 → 5 → 3	64.5
	3	3 → 4 → 5	58.7
		3 → 5 → 4	63.5
	4	3 → 4 → 5	58.7
		4 → 5 → 3	64.5
	5	5 → 4 → 3	64.5

When the precedence relationships implied by the criteria are compared to the optimal precedence relation (determined by complete enumeration of the possible precedence relations, Figure 6 - Figure 11 and Table 4 - 9), we find that using criterion 1, 2, and 4 provide the optimal project cost when the project deadline is 39, and criterion 1, 3, and 4 provide the optimal project cost when deadline is 35.

A second sample project network P_2 is depicted in Figure 12 and described in Table 11. The project deadline is assumed to be 14. The optimal schedule (ignoring resource constraints) is given in Table 12.

Assume that activity 2 and activity 3 compete for the same critical resource. An evaluation of the five precedence criteria is summarized in Table 13. In Table 13 the optimal project cost is 36 corresponding to using criteria 2, 3, or 4. Also assume that activity 4 and activity 5 compete for the same critical resources. The resolution of the conflict of activity 4 and activity 5 by the five precedence criteria is summarized in Table 14. In Table 14 the optimal project cost is 43 corresponding to using criteria 3 or 5.

Figure 12. Project Network for P_2

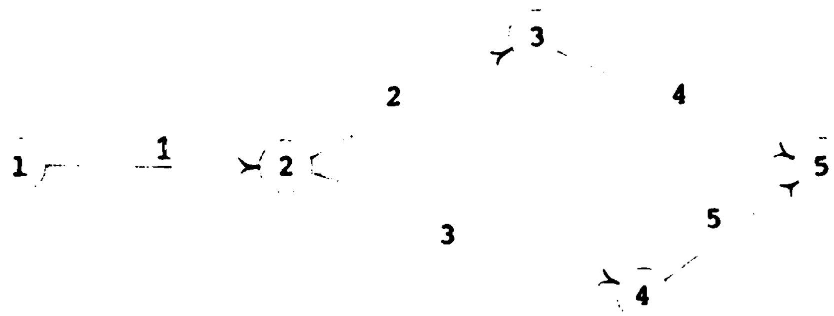

Table 11. Activity Costs and Durations for P_2

Activity	Origin	Terminal	Number of Different Completion Times	Time and Cost	Segments		
1	1	2	3	Time	3	5	7
				Cost	8	4	3
2	2	3	2	Time	4	4	
				Cost	8	8	
3	2	4	2	Time	2	6	
				Cost	8	3	
4	3	5	3	Time	3	5	7
				Cost	9	6	2
5	4	5	3	Time	4	8	12
				Cost	15	7	3

Table 12. Optimal Project Schedule for P_2 Ignoring Resource Constraints (Project Deadline = 14.)

Activity Number	Activity Duration	Activity Cost
1	3	8
2	4	8
3	3	6.75
4	7	2
5	8	7
Project Cost		31.75

Table 13. Precedence Relationship and Project Cost Corresponding to Criteria (1) - (5) When Activity 2 and 3 Conflict and Project Deadline is 14 in P_2

Criteria	Precedence Relationship	Project Cost
1	2 → 3	39
	3 → 2	36
2	3 → 2	36
3	3 → 2	36
4	3 → 2	36
5	2 → 3	39

Table 14. Precedence Relationship and Project Cost Corresponding to Criteria (1) - (5) When Activity 4 and 5 Conflict and Project Deadline is 14 in P_2

Criteria	Precedence Relationship	Project Cost
1	5 → 4	45
2	5 → 4	45
3	5 → 4	45
	4 → 5	43
4	5 → 4	45
5	4 → 5	43

The third sample project network P_3 is depicted in Figure 13 and described in Table 15. If the project completion time is 30, the optimal project schedule (ignoring resources constraints) is given in Table 16.

Assume that activity 3 and activity 4 compete for the same critical resource. An evaluation of the five precedence criteria is summarized in Table 17. In Table 17, the optimal project cost is 20.67 corresponding to criteria 1, 2, and 4 when the project deadline is 30.

Assume that activity 4 and activity 5 compete for the same critical resources. The results of the resolution of the resource conflict between activity 4 and activity 5 by the five precedence criteria are given in Table 18. In Table 18 the optimal project cost is 46 corresponding to criteria 1, 2, 3, 4, and 5 when project deadline is 30.

Figure 13. Project Network for P_3

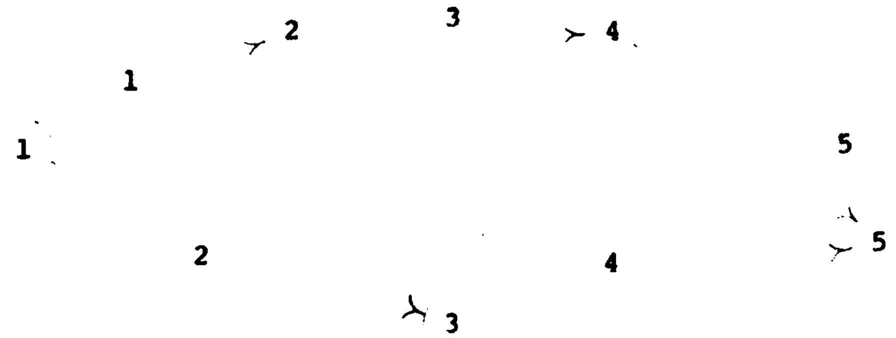

Table 15. Activity Costs and Duration for P_3

Activity Number	Origin Node	Terminal Node	Number of Different Completion Times	Time and Cost	Segments			
1	1	2	2	time	3	7		
				cost	8	3		
2	1	3	3	time	4	8	12	
				cost	16	8	4	
3	2	4	4	time	5	8	10	16
				cost	16	10	6	4
4	3	5	2	time	4	7		
				cost	10	5	9	
5	4	5	3	time	3	6	4	
				cost	12	8		

Table 16. Optimal Project Schedule for P_3
Ignoring Resource Constraint
(Project Deadline = 30.)

Activity Number	Activity Duration	Activity Cost
1	7	3
2	12	4
3	14	4.3
4	7	5
5	9	4

Table 17: Precedence Relationship and Project Cost Corresponding to Criteria (1) - (5) When Activity 3 and Activity 4 Conflict and Project Deadline is 30 in P_3

Criteria	Precedence Relationship	Project Cost
1	3 → 4	20.67
2	3 → 4	20.67
3	4 → 3	30
4	3 → 4	20.67
5	4 → 3	30

Table 18: Precedence Relationship and Project Cost Corresponding to Criteria (1) - (5) When Activity 4 and Activity 5 Conflict and Project Deadline is 30 in P_3

Criteria	Precedence Relationship	Project Cost
1	4 → 5	46
2	4 → 5	46
3	4 → 5	46
4	4 → 5	46
5	4 → 5	46

5. Checking the Usefulness of the Five Criteria

The usefulness of the precedence criteria in the six cases arising in the three experimental projects is given in Table 19.

Table 19. The Usefulness of the Precedence Criteria in the Three Experimental Projects*

Project Number		P_1		P_2		P_3	
Conflicting Activities		3,4,5	3,4,5	2,3	4,5	3,4	4,5
Project Deadline		39	35	14	14	30	30
Criteria	1	X	X			X	X
	2	X		X		X	X
	3		X	X	X		X
	4	X	X	X		X	X
	5				X		X

* the "X" denotes the criterion which yielded the minimum cost precedence relationship.

From Table 19 it is apparent that several criteria often yield the minimum cost schedule. However, criterion 4 which corresponds to ordering the conflicting activities so as to minimize the project completion time is most frequently optimal, 5 out of 6 cases. The optimal activity durations related to the one case in which criterion 4 failed are given in Table 20. From Table 20 it is apparent that in the one case where criteria 4 failed it did so because an activity, 3, not involved in the resource conflict had to have its duration substantially reduced (cost substantially increased) under the schedule corresponding to criterion 4 but not under the alternative schedule.

Table 20. Comparing the Project Schedules for a Project Deadline of 14

Activity	Original Schedule		Schedules Including Resource Constraint Criterion 4: 5 → 4		Other: 4 → 5	
	Time	Cost	Time	Cost	Time	Cost
1	3*	8	3*	8	3*	8
2	4	8	4	8	4*	8
3	3*	6.75	2*	8	6	3
4	7	2	5*	6	3*	9
5	8*	7	4*	15	4*	15
Total		31.75		45		43

*the time for an activity on the critical path

6. An "Easy" Way to Implement Criterion 4

Initially each activity's mean duration is chosen ignoring the resource constraints. If activities A and B have a resource conflict under this schedule, then criterion 4 implies that A should precede B, $A \rightarrow B$, if, for these original activity mean durations, the schedule with $A \rightarrow B$ has a smaller project completion time than the schedule with $B \rightarrow A$. The remainder of this section describes a relatively easy procedure for determining which activity order corresponds to the smaller project completion time for given activity mean durations.

The simplest case is when only activities, say A and B, compete for the same critical resource. The restructured network with $A \rightarrow B$ will have smaller project completion time if

$$O_A + T_B < O_B + T_A$$

where

O_i = the length of the longest path from the source to the <u>origin</u> node of activity i, and

T_i = the length of the longest path from the terminal node of activity i to the sink.

For example in the experimental project P_3 activities 3 and 4 compete for the same resource.

There

O_3 = 7 = duration of activity 1;

O_4 = 12 = duration of activity 2;

$T_3 = 9 =$ duration of activity 5;

$T_4 = 0$, since the terminal node of activity 4 is the sink.

Hence

$$O_3 + T_4 = 7 + 0 = 7 < O_4 + T_3 = 12 + 9 = 23$$

and activity 3 should precede activity 4. With the activity durations at their means and activity 3 preceding activity 4 the critical path contains activities 1, 3, and 5 and has completion time 30. With 4 → 3 the critical path contains 2, 3, 4, and 5 and has completion time 42.

For the more complicated situation where more than two activities compete for the same critical resource. We can use a similar strategy. Let activities A B and C compete for the same critical resource. If

$$O_A = \text{Min.} \quad \{O_A, O_B, O_C\}$$
$$T_B = \text{Min.} \quad \{T_A, T_B, T_C\} ,$$

then the project completion time is minimized when A → C → B. If

$$O_A = \text{Min} \quad \{O_A, O_B, O_C\}$$
$$T_A = \text{Min} \quad \{T_A, T_B, T_C\} ,$$

then we have to find Min $\{O_B, O_C\}$ and Min $\{T_B, T_C\}$, say O_B and T_C respectively. Then we find Min $\{O_A + T_C, O_B + T_A\}$. If $O_B + T_A$ is the minimum, then B → C → A leads to a minimum project completion time. If $O_A + T_C$ is the minimum, then A → B → C leads to a minimum project completion time.

For example in experimental project P_1 activities 3, 4 and 5 compete for the same critical resource when project deadline is 39. Here

$O_3 = 6 =$ duration of activity 1;

$O_4 = 9 =$ duration of activity 2;

$O_5 = 9 =$ duration of activity 2;

$O_3 = \text{Min}\ \{O_3,\ O_4\ ,\ O_5\}$;

$T_3 = 16 =$ duration of activities 6 and 9;

$T_4 = 17 =$ duration of activities 7 and 9;

$T_5 = 15 =$ duration of activity 8;

$T_5 = \text{Min}\ \{T_3,\ T_4,\ T_5\}$.

Hence $3 \rightarrow 4 \rightarrow 5$ leads to the minimum project completion time. In fact, checking all six possible precedence relations, we have the following:

precedence relation:	project completion time:
$3 \rightarrow 4 \rightarrow 5$	56
$3 \rightarrow 5 \rightarrow 4$	58
$4 \rightarrow 5 \rightarrow 3$	62
$4 \rightarrow 3 \rightarrow 5$	59
$5 \rightarrow 4 \rightarrow 3$	62
$5 \rightarrow 3 \rightarrow 4$	61 .

7. Concluding Remarks

When more than one activity in a project requires the same indivisible resource, the project schedule can resolve this resource constraint by specifying the order in which these activities are to be performed. Five heuristic criteria for determining this order have been considered. Although each criterion was sometimes useful, the minimum cost schedule for a given project deadline was most frequently found by determining the optimal schedule ignoring the resource constraints and then resolving any resource usage conflict by ordering the conflicting activities so as to minimize the project completion time.

This latter precedence criterion does not always yield a minimum cost schedule. This happens because the criterion chooses the activity ordering corresponding to the minimum project completion time for given activity mean durations and does not consider the cost of compressing this project completion time to meet a specified project deadline. The difficulty arises because the derivative of the activity cost with respect to the activity duration is not the same for all activities. Hence it may cost less to shorten one network's completion time by 5 units that to shorten another by 3 units.

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When more than one activity in a project requires the same indivisible resource, the project schedule can resolve this resource constraint by specifying the order in which these activities are to be performed. Several heuristic criteria for determining this order are considered. The minimum cost schedule for a given project deadline can often be found by determining the optimal schedule ignoring the resource

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constraints and then resolving any resource usage conflicts by ordering the conflicting activities so as to minimize the project completion time. A simple procedure for determining this latter order is given.

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